CHAPTER 11: Vibrations and Waves

Answers to Questions

1. The blades in an electric shaver vibrate, approximately in SHM.
The speakers in a stereo system vibrate, but usually in a very complicated way since many notes are being sounded at the same time.
A piano string vibrates when struck, in approximately SHM.
The pistons in a car engine oscillate, in approximately SHM.
The free end of a diving board oscillates after a diver jumps, in approximately SHM.

2. The acceleration of a simple harmonic oscillator is zero whenever the oscillating object is at the equilibrium position.

3. The motion of the piston can be approximated as simple harmonic. First of all, the piston will have a constant period while the engine is running at a constant speed. The speed of the piston will be zero at the extremes of its motion – the top and bottom of the stroke – which is the same as in simple harmonic motion. There is a large force exerted on the piston at one extreme of its motion – the combustion of the fuel mixture – and simple harmonic motion has the largest force at the extremes of the motion. Also, as the crankshaft moves in a circle, its component of motion in one dimension is transferred to the piston. It is similar to Fig. 11-6.

4. Since the real spring has mass, the mass that is moving is greater than the mass at the end of the spring. Since $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$, a larger mass means a smaller frequency. Thus the true frequency will be smaller than the "massless spring" approximation. And since the true frequency is smaller, the true period will be larger than the "massless spring" approximation. About 1/3 the mass of the spring contributes to the total mass value.

5. The maximum speed is given by $v_{max} = A\sqrt{k/m}$. Various combinations of changing *A*, *k*, and/or *m* can result in a doubling of the maximum speed. For example, if *k* and *m* are kept constant, then doubling the amplitude will double the maximum speed. Or, if *A* and *k* are kept constant, then reducing the mass to one-fourth its original value will double the maximum speed. Note that changing either *k* or *m* will also change the frequency of the oscillator, since $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$.

6. The scale reading will oscillate with damped oscillations about an equilibrium reading of 5.0 kg, with an initial amplitude of 5.0 kg (so the range of readings is initially from 0.0 kg and 10.0 kg). Due to friction in the spring and scale mechanism, the oscillation amplitude will decrease over time, eventually coming to rest at the 5.0 kg mark.

7. The period of a pendulum clock is inversely proportional to the square root of *g*, by Equation 11-11a, $T = 2\pi\sqrt{L/g}$. When taken to high altitude, the value of *g* will decrease (by a small amount), which means the period will increase. If the period is too long, the clock is running slow and so will lose time.

8. The tire swing approximates a simple pendulum. With a stopwatch, you can measure the period T of the tire swing, and then solve Equation 11-11a for the length, $L = \frac{gT^2}{4\pi^2}$.

9. To make the water "slosh", you must shake the water (and the pan) at the natural frequency for water waves in the pan. The water then is in resonance, or in a standing wave pattern, and the amplitude of oscillation gets large. That natural frequency is determined by the size of the pan – smaller pans will slosh at higher frequencies, corresponding to shorter wavelengths for the standing waves. The period of the shaking must be the same as the time it takes a water wave to make a "round trip" in the pan.

10. Some examples of resonance:
    Pushing a child on a playground swing – you always push at the frequency of the swing.
    Seeing a stop sign oscillating back and forth on a windy day.
    When singing in the shower, certain notes will sound much louder than others.
    Utility lines along the roadside can have a large amplitude due to the wind.
    Rubbing your finger on a wineglass and making it "sing".
    Blowing across the top of a bottle.
    A rattle in a car (see Question 11).

11. A rattle in a car is very often a resonance phenomenon. The car itself vibrates in many pieces, because there are many periodic motions occurring in the car – wheels rotating, pistons moving up and down, valves opening and closing, transmission gears spinning, driveshaft spinning, etc. There are also vibrations caused by irregularities in the road surface as the car is driven, such as hitting a hole in the road. If there is a loose part, and its natural frequency is close to one of the frequencies already occurring in the car's normal operation, then that part will have a larger than usual amplitude of oscillation, and it will rattle. This is why some rattles only occur at certain speeds when driving.

12. The frequency of a simple periodic wave is equal to the frequency of its source. The wave is created by the source moving the wave medium that is in contact with the source. If you have one end of a taut string in your hand, and you move your hand with a frequency of 2 Hz, then the end of the string in your hand will be moving at 2 Hz, because it is in contact with your hand. Then those parts of the medium that you are moving exert forces on adjacent parts of the medium and cause them to oscillate. Since those two portions of the medium stay in contact with each other, they also must be moving with the same frequency. That can be repeated all along the medium, and so the entire wave throughout the medium has the same frequency as the source.

13. The speed of the transverse wave is measuring how fast the wave disturbance moves along the cord. For a uniform cord, that speed is constant, and depends on the tension in the cord and the mass density of the cord. The speed of a tiny piece of the cord is measuring how fast the piece of cord moves perpendicularly to the cord, as the disturbance passes by. That speed is not constant – if a sinusoidal wave is traveling on the cord, the speed of each piece of the cord will be given by the speed relationship of a simple harmonic oscillator (Equation 11-9), which depends on the amplitude of the wave, the frequency of the wave, and the specific time of observation.

14. From Equation 11-19b, the fundamental frequency of oscillation for a string with both ends fixed is $f_1 = \frac{v}{2L}$. The speed of waves on the string is given by Equation 11-13, $v = \sqrt{\frac{F_T}{m/L}}$. Combining

these two relationships gives $f_1 = \frac{1}{2}\sqrt{\frac{F_T}{mL}}$. By wrapping the string with wire, the mass of the string can be greatly increased without changing the length or the tension of the string, and thus the string has a low fundamental frequency.

15. If you strike the horizontal rod vertically, you will create primarily transverse waves. If you strike the rod parallel to its length, you will create primarily longitudinal waves.

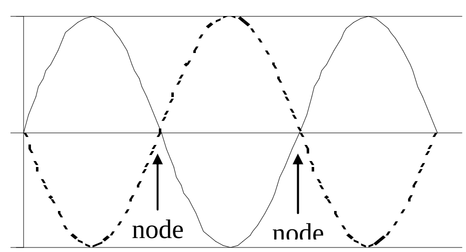
16. From Equation 11-14b, the speed of waves in a gas is given by $v = \sqrt{B/\rho}$. A decrease in the density due to a temperature increase therefore leads to a higher speed of sound. We expect the speed of sound to increase as temperature increases.

17. (*a*) Similar to the discussion in section 11-9 for spherical waves, as a circular wave expands, the circumference of the wave increases. For the energy in the wave to be conserved, as the circumference increases, the intensity has to decrease. The intensity of the wave is proportional to the square of the amplitude
    (*b*) The water waves will decrease in amplitude due to dissipation of energy from viscosity in the water (dissipative or frictional energy loss).

18. Assuming the two waves are in the same medium, then they will both have the same speed. Since $v = f\lambda$, the wave with the smaller wavelength will have twice the frequency of the other wave. From Equation 11-18, the intensity of wave is proportional to the square of the frequency of the wave. Thus the wave with the shorter wavelength will transmit 4 times as much energy as the other wave.

19. The frequency must stay the same because the media is continuous – the end of one section of cord is physically tied to the other section of cord. If the end of the first section of cord is vibrating up and down with a given frequency, then since it is attached to the other section of cord, the other section must vibrate at the same frequency. If the two pieces of cord did not move at the same frequency, they would not stay connected, and then the waves would not pass from one section to another.

20. The string could be touched at the location of a node without disturbing the motion, because the nodes do not move. A string vibrating in three segments has 2 nodes in addition to the ones at the ends. See the diagram.



21. The energy of a wave is not localized at one point, because the wave is not localized at one point, and so to talk about the energy "at a node" being zero is not really a meaningful statement. Due to the interference of the waves the total energy of the medium particles at the nodes points is zero, but the energy of the medium is not zero at points of the medium that are not nodes. In fact, the anti-node points have more energy than they would have if only one of the two waves were present.

22. A major distinction between energy transfer by particles and energy transfer by waves is that particles must travel in a straight line from one place to another in order to transfer energy, but waves can diffract around obstacles. For instance, sound can be heard around a corner, while you cannot throw a ball around a corner. So if a barrier is placed between the source of the energy and the

location where the energy is being received, and energy is still received in spite of the barrier, it is a good indication that the energy is being carried by waves. If the placement of the barrier stops the energy transfer, it could be that the energy transfer is being carried out by particles. It could also be that the energy transfer is being carried out with waves whose wavelength is much smaller than the dimensions of the barrier.

Solutions to Problems

1. The particle would travel four times the amplitude: from $x = A$ to $x = 0$ to $x = -A$ to $x = 0$ to $x = A$. So the total distance $= 4A = 4(0.18\text{ m}) = \boxed{0.72\text{ m}}$.

2. The spring constant is the ratio of applied force to displacement.

$$k = \frac{F}{x} = \frac{180\text{ N} - 75\text{ N}}{0.85\text{ m} - 0.65\text{ m}} = \frac{105\text{ N}}{0.20\text{ m}} = \boxed{5.3\times10^2\text{ N/m}}$$

3. The spring constant is found from the ratio of applied force to displacement.

$$k = \frac{F}{x} = \frac{mg}{x} = \frac{(68\text{ kg})(9.8\text{ m/s}^2)}{5\times10^{-3}\text{ m}} = 1.333\times10^5\text{ N/m}$$

The frequency of oscillation is found from the total mass and the spring constant.

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{1.333\times10^5\text{ N/m}}{1568\text{ kg}}} = 1.467\text{ Hz} \approx \boxed{1.5\text{ Hz}}$$

4. (*a*) The spring constant is found from the ratio of applied force to displacement.

$$k = \frac{F}{x} = \frac{mg}{x} = \frac{(2.7\text{ kg})(9.80\text{ m/s}^2)}{3.6\times10^{-2}\text{ m}} = 735\text{ N/m} \approx \boxed{7.4\times10^2\text{ N/m}}$$

(*b*) The amplitude is the distance pulled down from equilibrium, so $A = \boxed{2.5\times10^{-2}\text{ m}}$

The frequency of oscillation is found from the total mass and the spring constant.

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{735\text{ N/m}}{2.7\text{ kg}}} = 2.626\text{ Hz} \approx \boxed{2.6\text{ Hz}}$$

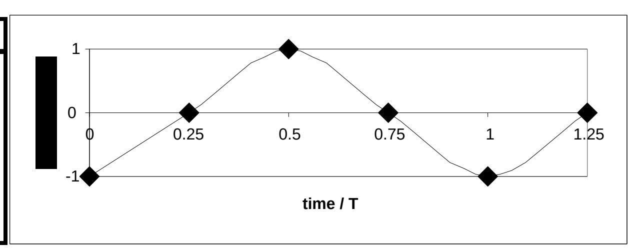
5. The spring constant is the same regardless of what mass is hung from the spring.

$$f = \frac{1}{2\pi}\sqrt{k/m} \rightarrow \sqrt{k}/2\pi = f\sqrt{m} = \text{constant} \rightarrow f_1\sqrt{m_1} = f_2\sqrt{m_2} \rightarrow$$

$$f_2 = f_1\sqrt{m_1/m_2} = (3.0\text{ Hz})\sqrt{0.60\text{ kg}/0.38\text{ kg}} = \boxed{3.8\text{ Hz}}$$

6. The table of data is shown, along with the smoothed graph. Every quarter of a period, the mass moves from an extreme point to the equilibrium. The graph resembles a cosine wave (actually, the opposite of a cosine wave).

time	position
0	- A
T /4	0
T/2	A
3T/4	0
T	- A
5T/4	0



7. The relationship between frequency, mass, and spring constant is $f=\frac{1}{2\pi}\sqrt{\frac{k}{m}}$.

(*a*) $f=\frac{1}{2\pi}\sqrt{\frac{k}{m}} \rightarrow k=4\pi^2 f^2 m=4\pi^2\left(4.0\text{ Hz}\right)^2\left(2.5\times10^{-4}\text{ kg}\right)=0.1579\text{ N/m}\approx\boxed{0.16\text{ N/m}}$

(*b*) $f=\frac{1}{2\pi}\sqrt{\frac{k}{m}}=\frac{1}{2\pi}\sqrt{\frac{0.1579\text{ N/m}}{5.0\times10^{-4}\text{ kg}}}=\boxed{2.8\text{ Hz}}$

8. The spring constant is the same regardless of what mass is attached to the spring.

$$f=\frac{1}{2\pi}\sqrt{\frac{k}{m}} \rightarrow \frac{k}{4\pi^2}=mf^2=\text{constant} \rightarrow m_1 f_1^2=m_2 f_1^2 \rightarrow$$

$$\left(m\text{ kg}\right)\left(0.88\text{ Hz}\right)^2=\left(m\text{ kg}+0.68\text{ kg}\right)\left(0.60\text{ Hz}\right)^2 \rightarrow m=\frac{\left(0.68\text{ kg}\right)\left(0.60\text{ Hz}\right)^2}{\left(0.88\text{ Hz}\right)^2-\left(0.60\text{ Hz}\right)^2}=\boxed{0.59\text{ kg}}$$

9. (*a*) At equilibrium, the velocity is its maximum.

$$v_{\text{max}}=\sqrt{\frac{k}{m}}A=\omega A=2\pi fA=2\pi\left(3\text{ Hz}\right)\left(0.13\text{ m}\right)=2.450\text{ m/s}\approx\boxed{2.5\text{ m/s}}$$

(*b*) From Equation (11-5), we find the velocity at any position.

$$v=\pm v_{\text{max}}\sqrt{1-\frac{x^2}{A^2}}=\pm\left(2.45\text{ m/s}\right)\sqrt{1-\frac{\left(0.10\text{ m}\right)^2}{\left(0.13\text{ m}\right)^2}}=\pm1.565\text{ m/s}\approx\boxed{\pm1.6\text{ m/s}}$$

(*c*) $E_{\text{total}}=\frac{1}{2}mv_{\text{max}}^2=\frac{1}{2}\left(0.60\text{ kg}\right)\left(2.45\text{ m/s}\right)^2=1.801\text{ J}\approx\boxed{1.8\text{ J}}$

(*d*) Since the object has a maximum displacement at t = 0, the position will be described by the cosine function.

$$x=\left(0.13\text{ m}\right)\cos\left(2\pi\left(3.0\text{ Hz}\right)t\right) \rightarrow \boxed{x=\left(0.13\text{ m}\right)\cos\left(6.0\pi t\right)}$$

10. The relationship between the velocity and the position of a SHO is given by Equation (11-5). Set that expression equal to half the maximum speed, and solve for the displacement.

$$v=\pm v_{\text{max}}\sqrt{1-x^2/A^2}=\tfrac{1}{2}v_{\text{max}} \rightarrow \pm\sqrt{1-x^2/A^2}=\tfrac{1}{2} \rightarrow 1-x^2/A^2=\tfrac{1}{4} \rightarrow x^2/A^2=\tfrac{3}{4} \rightarrow$$

$$\boxed{x=\pm\sqrt{3}A/2\approx0.866A}$$

11. Since $F=-kx=ma$ for an object attached to a spring, the acceleration is proportional to the displacement (although in the opposite direction), as $a=-x\,k/m$. Thus the acceleration will have half its maximum value where the displacement has half its maximum value, at $\boxed{\pm\frac{1}{2}x_0}$

12. The spring constant can be found from the stretch distance corresponding to the weight suspended on the spring.

$$k=\frac{F}{x}=\frac{mg}{x}=\frac{\left(2.62\text{ kg}\right)\left(9.80\text{ m/s}^2\right)}{0.315\text{ m}}=81.5\text{ N/m}$$

After being stretched further and released, the mass will oscillate. It takes one-quarter of a period for the mass to move from the maximum displacement to the equilibrium position.

$$\tfrac{1}{4}T = \tfrac{1}{4}2\pi\sqrt{m/k} = \frac{\pi}{2}\sqrt{\frac{2.62\text{ kg}}{81.5\text{ N/m}}} = \boxed{0.282\text{ s}}$$

13. (*a*) The total energy of an object in SHM is constant. When the position is at the amplitude, the speed is zero. Use that relationship to find the amplitude.

$$E_{\text{tot}} = \tfrac{1}{2}mv^2 + \tfrac{1}{2}kx^2 = \tfrac{1}{2}kA^2 \quad\rightarrow$$

$$A = \sqrt{\frac{m}{k}v^2 + x^2} = \sqrt{\frac{3.0\text{ kg}}{280\text{ N/m}}(0.55\text{ m/s})^2 + (0.020\text{ m})^2} = 6.034\times10^{-2}\text{ m} \approx \boxed{6.0\times10^{-2}\text{ m}}$$

(*b*) Again use conservation of energy. The energy is all kinetic energy when the object has its maximum velocity.

$$E_{\text{tot}} = \tfrac{1}{2}mv^2 + \tfrac{1}{2}kx^2 = \tfrac{1}{2}kA^2 = \tfrac{1}{2}mv_{\text{max}}^2 \quad\rightarrow$$

$$v_{\text{max}} = A\sqrt{\frac{k}{m}} = \left(6.034\times10^{-2}\text{ m}\right)\sqrt{\frac{280\text{ N/m}}{3.0\text{ kg}}} = 0.5829\text{ m/s} \approx \boxed{0.58\text{ m/s}}$$

14. The spring constant is found from the ratio of applied force to displacement.

$$k = \frac{F}{x} = \frac{80.0\text{ N}}{0.200\text{ m}} = 4.00\times10^2\text{ N/m}$$

Assuming that there are no dissipative forces acting on the ball, the elastic potential energy in the loaded position will become kinetic energy of the ball.

$$E_i = E_f \quad\rightarrow\quad \tfrac{1}{2}kx_{\text{max}}^2 = \tfrac{1}{2}mv_{\text{max}}^2 \quad\rightarrow\quad v_{\text{max}} = x_{\text{max}}\sqrt{\frac{k}{m}} = (0.200\text{ m})\sqrt{\frac{4.00\times10^2\text{ N/m}}{0.180\text{ kg}}} = \boxed{9.43\text{ m/s}}$$

15. (*a*) The work done to compress a spring is stored as potential energy.

$$W = \tfrac{1}{2}kx^2 \quad\rightarrow\quad k = \frac{2W}{x^2} = \frac{2(3.0\text{ J})}{(0.12\text{ m})^2} = 416.7\text{ N/m} \approx \boxed{4.2\times10^2\text{ N/m}}$$

(*b*) The distance that the spring was compressed becomes the amplitude of its motion. The maximum acceleration is given by $a_{\text{max}} = \frac{k}{m}A$. Solve this for the mass.

$$a_{\text{max}} = \frac{k}{m}A \quad\rightarrow\quad m = \frac{k}{a_{\text{max}}}A = \left(\frac{4.167\times10^2\text{ N/m}}{15\text{ m/s}^2}\right)(0.12\text{ m}) = 3.333\text{ kg} \approx \boxed{3.3\text{ kg}}$$

16. The general form of the motion is $x = A\cos\omega t = 0.45\cos 6.40t$.

(*a*) The amplitude is $A = x_{\text{max}} = \boxed{0.45\text{ m}}$.

(*b*) The frequency is found by $\omega = 2\pi f = 6.40\text{ s}^{-1} \quad\rightarrow\quad f = \frac{6.40\text{ s}^{-1}}{2\pi} = 1.019\text{ Hz} \approx \boxed{1.02\text{ Hz}}$

(*c*) The total energy is given by

$$E_{\text{total}} = \tfrac{1}{2}mv_{\text{max}}^2 = \tfrac{1}{2}m(\omega A)^2 = \tfrac{1}{2}(0.60\text{ kg})\left[(6.40\text{ s}^{-1})(0.45\text{ m})\right]^2 = 2.488\text{ J} \approx \boxed{2.5\text{ J}}$$

(*d*) The potential energy is given by

$$E_{\text{potential}} = \tfrac{1}{2}kx^2 = \tfrac{1}{2}m\omega^2x^2 = \tfrac{1}{2}(0.60\text{ kg})(6.40\text{ s}^{-1})^2(0.30\text{ m})^2 = 1.111\text{ J} \approx \boxed{1.1\text{ J}}$$

The kinetic energy is given by

$$E_{\text{kinetic}} = E_{\text{total}} - E_{\text{potential}} = 2.488\text{ J} - 1.111\text{ J} = 1.377\text{ J} \approx \boxed{1.4\text{ J}}$$

17. If the energy of the SHO is half potential and half kinetic, then the potential energy is half the total energy. The total energy is the potential energy when the displacement has the value of the amplitude.

$$E_{\text{pot}} = \tfrac{1}{2}E_{\text{tot}} \quad\rightarrow\quad \tfrac{1}{2}kx^2 = \tfrac{1}{2}\left(\tfrac{1}{2}kA^2\right) \quad\rightarrow\quad \boxed{x = \pm\frac{1}{\sqrt{2}}A \approx \pm 0.707A}$$

18. If the frequencies and masses are the same, then the spring constants for the two vibrations are the same. The total energy is given by the maximum potential energy.

$$\frac{E_1}{E_2} = \frac{\tfrac{1}{2}kA_1^2}{\tfrac{1}{2}kA_2^2} = \left(\frac{A_1}{A_2}\right)^2 = 7.0 \quad\rightarrow\quad \frac{A_1}{A_2} = \sqrt{7.0} = \boxed{2.6}$$

19. (*a*) The general equation for SHM is Equation (11-8c), $y = A\cos\left(2\pi t/T\right)$. For the pumpkin,

$$\boxed{y = \left(0.18\text{ m}\right)\cos\left(\frac{2\pi t}{0.65\text{ s}}\right)}.$$

(*b*) The time to return back to the equilibrium position is one-quarter of a period.

$$t = \tfrac{1}{4}T = \tfrac{1}{4}\left(0.65\text{ s}\right) = \boxed{0.16\text{ s}}$$

(*c*) The maximum speed is given by the angular frequency times the amplitude.

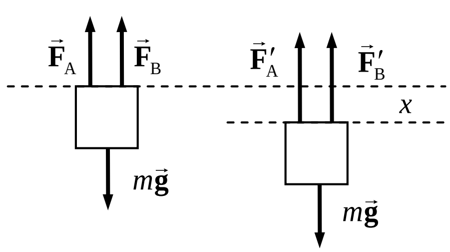
$$v_{\max} = \omega A = \frac{2\pi}{T}A = \frac{2\pi}{0.65\text{ s}}\left(0.18\text{ m}\right) = \boxed{1.7\text{ m/s}}$$

(*d*) The maximum acceleration is given by

$$a_{\max} = \omega^2 A = \left(\frac{2\pi}{T}\right)^2 A = \frac{4\pi^2}{\left(0.65\text{ s}\right)^2}\left(0.18\text{ m}\right) = \boxed{17\text{ m/s}^2}.$$

The maximum acceleration is first attained at the release point of the pumpkin.

20. Consider the first free-body diagram for the block while it is at equilibrium, so that the net force is zero. Newton's 2nd law for vertical forces, choosing up as positive, gives this.



$$\sum F_y = F_A + F_B - mg = 0 \quad\rightarrow\quad F_A + F_B = mg$$

Now consider the second free-body diagram, in which the block is displaced a distance x from the equilibrium point. Each upward force will have increased by an amount $-kx$, since $x < 0$. Again write Newton's 2nd law for vertical forces.

$$\sum F_y = F_{net} = F'_A + F'_B - mg = F_A - kx + F_B - kx - mg = -2kx + \left(F_A + F_B - mg\right) = -2kx$$

This is the general form of a restoring force that produces SHM, with an effective spring constant of $2k$. Thus the frequency of vibration is as follows.

$$f = \frac{1}{2\pi}\sqrt{k_{\text{effective}}/m} = \boxed{\frac{1}{2\pi}\sqrt{\frac{2k}{m}}}$$

21. The equation of motion is $x = 0.38 \sin 6.50t = A \sin \omega t$.

(*a*) The amplitude is $A = x_{max} = \boxed{0.38 \text{ m}}$.

(*b*) The frequency is found by $\omega = 2\pi f = 6.50 \text{ s}^{-1} \rightarrow f = \dfrac{6.50 \text{ s}^{-1}}{2\pi} = \boxed{1.03 \text{ Hz}}$

(*c*) The period is the reciprocal of the frequency. $T = 1/f = 1/1.03 \text{ Hz} = \boxed{0.967 \text{ s}}$.

(*d*) The total energy is given by

$$E_{total} = \tfrac{1}{2} m v_{max}^2 = \tfrac{1}{2} m (\omega A)^2 = \tfrac{1}{2}(0.300 \text{ kg})\left[(6.50 \text{ s}^{-1})(0.38 \text{ m})\right]^2 = 0.9151 \text{ J} \approx \boxed{0.92 \text{ J}}.$$

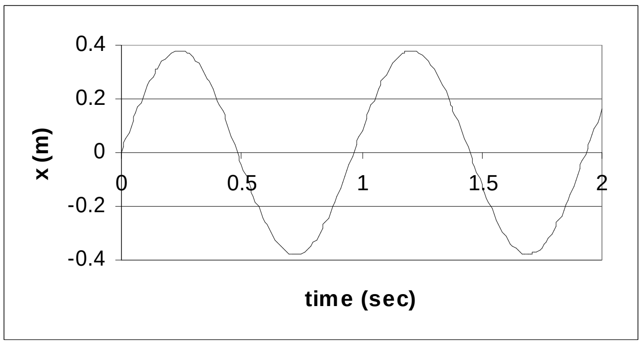
(*e*) The potential energy is given by

$$E_{potential} = \tfrac{1}{2} k x^2 = \tfrac{1}{2} m \omega^2 x^2 = \tfrac{1}{2}(0.300 \text{ kg})(6.50 \text{ s}^{-1})^2 (0.090 \text{m})^2 = 0.0513 \text{ J} \approx \boxed{5.1 \times 10^{-2} \text{ J}}.$$

The kinetic energy is given by

$$E_{kinetic} = E_{total} - E_{potential} = 0.9151 \text{J} - 0.0513 \text{J} = 0.8638 \text{J} \approx \boxed{0.86 \text{ J}}.$$

(*f*)



22. (*a*) For A, the amplitude is $A_A = \boxed{2.5 \text{ m}}$. For B, the amplitude is $A_B = \boxed{3.5 \text{ m}}$.

(*b*) For A, the frequency is 1 cycle every 4.0 seconds, so $f_A = \boxed{0.25 \text{ Hz}}$. For B, the frequency is 1 cycle every 2.0 seconds, so $f_B = \boxed{0.50 \text{ Hz}}$.

(*c*) For C, the period is $T_A = \boxed{4.0 \text{ s}}$. For B, the period is $T_B = \boxed{2.0 \text{ s}}$

(*d*) Object A has a displacement of 0 when $t = 0$, so it is a sine function.

$$x_A = A_A \sin(2\pi f_A t) \rightarrow \boxed{x_A = (2.5 \text{ m}) \sin\left(\frac{\pi}{2} t\right)}$$

Object B has a maximum displacement when $t = 0$, so it is a cosine function.

$$x_B = A_B \cos(2\pi f_B t) \rightarrow \boxed{x_B = (3.5 \text{ m}) \cos(\pi t)}$$

$\boxed{23.}$ (*a*) Find the period and frequency from the mass and the spring constant.

$$T = 2\pi\sqrt{m/k} = 2\pi\sqrt{0.755 \text{ kg}/(124 \text{ N/m})} = \boxed{0.490 \text{ s}} \qquad f = 1/T = 1/(0.490 \text{ s}) = \boxed{2.04 \text{ Hz}}$$

(*b*) The initial speed is the maximum speed, and that can be used to find the amplitude.

$$v_{max} = A\sqrt{k/m} \rightarrow A = v_{max}\sqrt{m/k} = (2.96 \text{ m/s})\sqrt{0.755 \text{ kg}/(124 \text{ N/m})} = \boxed{0.231 \text{ m}}$$

(*c*) The maximum acceleration can be found from the mass, spring constant, and amplitude

$$a_{max} = Ak/m = (0.231 \text{ m})(124 \text{ N/m})/(0.755 \text{ kg}) = \boxed{37.9 \text{ m/s}^2}$$

(*d*) Because the mass started at the equilibrium position of $x = 0$, the position function will be proportional to the sine function.

$$x = (0.231\text{ m})\sin\left[2\pi(2.04\text{ Hz})t\right] \rightarrow \boxed{x = (0.231\text{ m})\sin(4.08\pi t)}$$

(*e*) The maximum energy is the kinetic energy that the object has when at the equilibrium position.

$$E = \tfrac{1}{2}mv_{\text{max}}^2 = \tfrac{1}{2}(0.755\text{ kg})(2.96\text{ m/s})^2 = \boxed{3.31\text{ J}}$$

24. We assume that downward is the positive direction of motion. For this motion, we have $k = 305\text{ N/m}$, $A = 0.280\text{ m}$, $m = 0.260\text{ kg}$ and $\omega = \sqrt{k/m} = \sqrt{305\text{ N/m}/0.260\text{ kg}} = 34.250\text{ rad/s}$.

(*a*) Since the mass has a zero displacement and a positive velocity at $t = 0$, the equation is a sine function.

$$\boxed{y(t) = (0.280\text{ m})\sin\left[(34.3\text{ rad/s})t\right]}$$

(*b*) The period of oscillation is given by $T = \dfrac{2\pi}{\omega} = \dfrac{2\pi}{34.25\text{ rad/s}} = 0.18345\text{ s}$. The spring will have its maximum extension at times given by the following.

$$t_{\text{max}} = \frac{T}{4} + nT = \boxed{4.59\times10^{-2}\text{ s} + n(0.183\text{ s}),\ n = 0,1,2,\cdots}$$

The spring will have its minimum extension at times given by the following.

$$t_{\text{min}} = \frac{3T}{4} + nT = \boxed{1.38\times10^{-1}\text{ s} + n(0.183\text{ s}),\ n = 0,1,2,\cdots}$$

25. If the block is displaced a distance *x* to the right in the diagram, then spring # 1 will exert a force $F_1 = -k_1 x$, in the opposite direction to *x*. Likewise, spring # 2 will exert a force $F_2 = -k_2 x$, in the same direction as F_1. Thus the net force on the block is $F = F_1 + F_2 = -k_1 x - k_2 x = -(k_1 + k_2)x$. The effective spring constant is thus $k = k_1 + k_2$, and the period is given by $T = 2\pi\sqrt{\dfrac{m}{k}} = \boxed{2\pi\sqrt{\dfrac{m}{k_1 + k_2}}}$.

26. The energy of the oscillator will be conserved after the collision. Thus

$$E = \tfrac{1}{2}kA^2 = \tfrac{1}{2}(m + M)v_{\text{max}}^2 \rightarrow v_{\text{max}} = A\sqrt{k/(m+M)}$$

This speed is the speed that the block and bullet have immediately after the collision. Linear momentum in one dimension will have been conserved during the collision, and so the initial speed of the bullet can be found.

$$p_{\text{before}} = p_{\text{after}} \rightarrow mv_o = (m + M)v_{\text{max}}$$

$$v_o = \frac{m+M}{m}A\sqrt{\frac{k}{m+M}} = \frac{6.25\times10^{-1}\text{kg}}{2.5\times10^{-2}\text{kg}}(2.15\times10^{-1}\text{m})\sqrt{\frac{7.70\times10^{3}\text{ N/m}}{6.25\times10^{-1}\text{kg}}} = \boxed{597\text{ m/s}}$$

27. The period of the jumper's motion is $T = \dfrac{38.0\text{ s}}{8\text{ cycles}} = 4.75\text{ s}$. The spring constant can then be found from the period and the jumper's mass.

$$T = 2\pi\sqrt{\frac{m}{k}} \rightarrow k = \frac{4\pi^2 m}{T^2} = \frac{4\pi^2(65.0\text{ kg})}{(4.75\text{ s})^2} = 113.73\text{ N/m} \approx \boxed{114\text{ N/m}}$$

The stretch of the bungee cord needs to provide a force equal to the weight of the jumper when he is at the equilibrium point.

$$k\Delta x = mg \quad \rightarrow \quad \Delta x = \frac{mg}{k} = \frac{(65.0\text{ kg})(9.80\text{ m/s}^2)}{113.73\text{ N/m}} = 5.60\text{ m}$$

Thus the unstretched bungee cord must be $25.0\text{ m} - 5.60\text{ m} = \boxed{19.4\text{ m}}$

28. (*a*) The period is given by $T = \dfrac{60\text{ s}}{36\text{ cycles}} = \boxed{1.7\text{ s/cycle}}$.

(*b*) The frequency is given by $f = \dfrac{36\text{ cycles}}{60\text{ s}} = \boxed{0.60\text{ Hz}}$.

29. The period of a pendulum is given by $T = 2\pi\sqrt{L/g}$. Solve for the length using a period of 2.0 seconds.

$$T = 2\pi\sqrt{L/g} \quad \rightarrow \quad L = \frac{T^2 g}{4\pi^2} = \frac{(2.0\text{ s})^2(9.8\text{ m/s}^2)}{4\pi^2} = \boxed{0.99\text{ m}}$$

30. The period of a pendulum is given by $T = 2\pi\sqrt{L/g}$. The length is assumed to be the same for the pendulum both on Mars and on Earth.

$$T = 2\pi\sqrt{L/g} \quad \rightarrow \quad \frac{T_{\text{Mars}}}{T_{\text{Earth}}} = \frac{2\pi\sqrt{L/g_{\text{Mars}}}}{2\pi\sqrt{L/g_{\text{Earth}}}} = \sqrt{\frac{g_{\text{Earth}}}{g_{\text{Mars}}}} \quad \rightarrow$$

$$T_{\text{Mars}} = T_{\text{Earth}}\sqrt{\frac{g_{\text{Earth}}}{g_{\text{Mars}}}} = (0.80\text{ s})\sqrt{\frac{1}{0.37}} = \boxed{1.3\text{ s}}$$

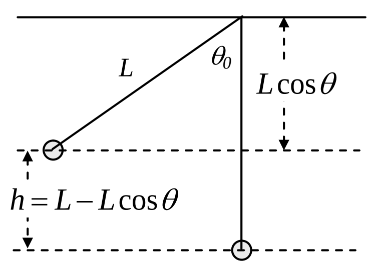
31. The period of a pendulum is given by $T = 2\pi\sqrt{L/g}$.

(*a*) $T = 2\pi\sqrt{L/g} = 2\pi\sqrt{\dfrac{0.80\text{ m}}{9.8\text{ m/s}^2}} = \boxed{1.8\text{ s}}$

(*b*) If the pendulum is in free fall, there is no tension in the string supporting the pendulum bob, and so no restoring force to cause oscillations. Thus there will be no period – the pendulum will not oscillate and so no period can be defined.

32. (*a*) The frequency can be found from the length of the pendulum, and the acceleration due to gravity.

$$f = \frac{1}{2\pi}\sqrt{\frac{g}{L}} = \frac{1}{2\pi}\sqrt{\frac{9.80\text{ m/s}^2}{0.760\text{ m}}} = 0.57151\text{ Hz} \approx \boxed{0.572\text{ Hz}}$$



(*b*) To find the speed at the lowest point, use the conservation of energy relating the lowest point to the release point of the pendulum. Take the lowest point to be the zero level of gravitational potential energy.

$$E_{\text{top}} = E_{\text{bottom}} \quad \rightarrow \quad KE_{\text{top}} + PE_{\text{top}} = KE_{\text{bottom}} + PE_{\text{bottom}}$$

$$0 + mg\left(L - L\cos\theta_o\right) = \tfrac{1}{2}mv_{\text{bottom}}^2 + 0$$

$$v_{\text{bottom}} = \sqrt{2gL\left(1-\cos\theta_o\right)} = \sqrt{2\left(9.80\,\text{m/s}^2\right)\left(0.760\text{m}\right)\left(1-\cos 12.0^\circ\right)} = \boxed{0.571\ \text{m/s}}$$

(*c*) The total energy can be found from the kinetic energy at the bottom of the motion.

$$E_{\text{total}} = \tfrac{1}{2}mv_{\text{bottom}}^2 = \tfrac{1}{2}\left(0.365\text{ kg}\right)\left(0.571\,\text{m/s}\right)^2 = \boxed{5.95\times10^{-2}\,\text{J}}$$

33. There are $\left(24\text{ h}\right)\left(60\,\text{min/h}\right)\left(60\,\text{s/min}\right) = 86,400\text{ s}$ in a day. The clock should make one cycle in exactly two seconds (a "tick" and a "tock"), and so the clock should make 43,200 cycles per day. After one day, the clock in question is 30 seconds slow, which means that it has made 15 less cycles than required for precise timekeeping. Thus the clock is only making 43,185 cycles in a day.

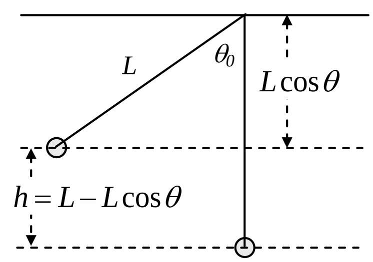
Accordingly, the period of the clock must be decreased by a factor $\dfrac{43{,}185}{43{,}200}$.

$$T_{\text{new}} = \frac{43{,}185}{43{,}200}T_{\text{old}} \quad\rightarrow\quad 2\pi\sqrt{L_{\text{new}}/g} = \left(\frac{43{,}185}{43{,}200}\right)2\pi\sqrt{L_{\text{old}}/g} \quad\rightarrow$$

$$L_{\text{new}} = \left(\frac{43{,}185}{43{,}200}\right)^2 L_{\text{old}} = \left(\frac{43{,}185}{43{,}200}\right)^2\left(0.9930\text{ m}\right) = 0.9923\text{ m}$$

Thus the pendulum should be $\boxed{\text{shortened by 0.7 mm}}$.

34. Use energy conservation to relate the potential energy at the maximum height of the pendulum to the kinetic energy at the lowest point of the swing. Take the lowest point to be the zero location for gravitational potential energy. See the diagram.



$$E_{\text{top}} = E_{\text{bottom}} \quad\rightarrow\quad KE_{\text{top}} + PE_{\text{top}} = KE_{\text{bottom}} + PE_{\text{bottom}} \quad\rightarrow$$

$$0 + mgh = \tfrac{1}{2}mv_{\text{max}}^2 \quad\rightarrow\quad v_{\text{max}} = \sqrt{2gh} = \boxed{\sqrt{2gL\left(1-\cos\theta_o\right)}}$$

35. The equation of motion for an object in SHM that has the maximum displacement at $t = 0$ is given by $x = A\cos\left(2\pi f\,t\right)$. For a pendulum, $x = L\theta$ and so $x_{\text{max}} = A = L\theta_{\text{max}}$, where θ must be measured in radians. Thus the equation for the pendulum's angular displacement is

$$L\theta = L\theta_{\text{max}}\cos\left(2\pi f\,t\right) \quad\rightarrow\quad \theta = \theta_{\text{max}}\cos\left(2\pi f\,t\right)$$

If both sides of the equation are multiplied by $180^\circ/\pi$ rad, then the angles can be measured in degrees. Thus the angular displacement of the pendulum can be written as below. Please note that the argument of the cosine function is still in radians.

$$\theta^\circ = \theta^\circ_{\text{max}}\cos\left(2\pi ft\right) = 15^\circ\cos\left(5.0\pi t\right)$$

(*a*) $\theta^\circ\left(t = 0.25\text{ s}\right) = 15^\circ\cos\left(5.0\pi\left(0.25\right)\right) = \boxed{-11^\circ}$

(*b*) $\theta^\circ\left(t = 1.6\text{ s}\right) = 15^\circ\cos\left(5.0\pi\left(1.6\right)\right) = \boxed{15^\circ}$ (here the time is exactly 4 periods)

(*c*) $\theta^\circ\left(t = 500\text{ s}\right) = 15^\circ\cos\left(5.0\pi\left(500\right)\right) = \boxed{15^\circ}$ (here the time is exactly 1250 periods)

36. The wave speed is given by $v = \lambda f$. The period is 3.0 seconds, and the wavelength is 6.5 m.

$$v = \lambda f = \lambda/T = \left(6.5\text{ m}\right)/\left(3.0\text{ s}\right) = \boxed{2.2\,\text{m/s}}$$

37. The distance between wave crests is the wavelength of the wave.

$$\lambda = v/f = 343\text{ m/s}/262\text{ Hz} = \boxed{1.31\text{ m}}$$

38. To find the wavelength, use $\lambda = v/f$.

AM: $\lambda_1 = \frac{v}{f_1} = \frac{3.00\times10^8\text{ m/s}}{550\times10^3\text{ Hz}} = 545\text{ m}$ $\quad \lambda_2 = \frac{v}{f_2} = \frac{3.00\times10^8\text{ m/s}}{1600\times10^3\text{ Hz}} = 188\text{ m}$ $\quad \boxed{\text{AM: 190 m to 550 m}}$

FM: $\lambda_1 = \frac{v}{f_1} = \frac{3.00\times10^8\text{ m/s}}{88.0\times10^6\text{ Hz}} = 3.41\text{ m}$ $\quad \lambda_2 = \frac{v}{f_2} = \frac{3.00\times10^8\text{ m/s}}{108\times10^6\text{ Hz}} = 2.78\text{ m}$ $\quad \boxed{\text{FM: 2.78 m to 3.41 m}}$

39. The elastic and bulk moduli are taken from Table 9-1 in chapter 9. The densities are taken from Table 10-1 in chapter 10.

(*a*) For water: $v = \sqrt{B/\rho} = \sqrt{\frac{2.0\times10^9\text{ N/m}^2}{1.00\times10^3\text{ kg/m}^3}} = \boxed{1.4\times10^3\text{ m/s}}$

(*b*) For granite: $v = \sqrt{E/\rho} = \sqrt{\frac{45\times10^9\text{ N/m}^2}{2.7\times10^3\text{ kg/m}^3}} = \boxed{4.1\times10^3\text{ m/s}}$

(*c*) For steel: $v = \sqrt{E/\rho} = \sqrt{\frac{200\times10^9\text{ N/m}^2}{7.8\times10^3\text{ kg/m}^3}} = \boxed{5.1\times10^3\text{ m/s}}$

40. The speed of a longitudinal wave in a solid is given by $v = \sqrt{E/\rho}$. Call the density of the less dense material ρ_1, and the density of the more dense material ρ_2. The less dense material will have the higher speed, since the speed is inversely proportional to the square root of the density.

$$\frac{v_1}{v_2} = \frac{\sqrt{E/\rho_1}}{\sqrt{E/\rho_2}} = \sqrt{\frac{\rho_2}{\rho_1}} = \sqrt{2} = \boxed{1.41}$$

41. To find the time for a pulse to travel from one end of the cord to the other, the velocity of the pulse on the cord must be known. For a cord under tension, we have $v = \sqrt{\frac{F_T}{m/L}}$.

$$v = \frac{\Delta x}{\Delta t} = \sqrt{\frac{F_T}{m/L}} \rightarrow \Delta t = \frac{\Delta x}{\sqrt{\frac{F_T}{m/L}}} = \frac{28\text{ m}}{\sqrt{\frac{150\text{ N}}{(0.65\text{ kg})/(28\text{ m})}}} = \boxed{0.35\text{ s}}$$

42. (*a*) The speed of the pulse is given by

$$v = \frac{\Delta x}{\Delta t} = \frac{2(620\text{ m})}{16\text{ s}} = 77.5\text{ m/s} \approx \boxed{78\text{ m/s}}$$

(*b*) The tension is related to the speed of the pulse by $v = \sqrt{\frac{F_T}{m/L}}$. The mass per unit length of the cable can be found from its volume and density.

$$\rho = \frac{m}{V} = \frac{m}{\pi (d/2)^2 L} \;\rightarrow\; \frac{m}{L} = \pi\rho\left(\frac{d}{2}\right)^2 = \pi\left(7.8\times10^3\ \mathrm{kg/m^3}\right)\left(\frac{1.5\times10^{-2}\ \mathrm{m}}{2}\right)^2 = 1.378\ \mathrm{kg/m}$$

$$v = \sqrt{\frac{F_T}{m/L}} \;\rightarrow\; F_T = v^2\frac{m}{L} = (77.5\ \mathrm{m/s})^2(1.378\ \mathrm{kg/m}) = \boxed{8.3\times10^3\ \mathrm{N}}$$

43. The speed of the water wave is given by $v = \sqrt{B/\rho}$, where B is the bulk modulus of water, from Table 9-1, and ρ is the density of sea water, from Table 10-1. The wave travels twice the depth of the ocean during the elapsed time.

$$v = \frac{2L}{t} \;\rightarrow\; L = \frac{vt}{2} = \frac{t}{2}\sqrt{\frac{B}{\rho}} = \frac{3.0\ \mathrm{s}}{2}\sqrt{\frac{2.0\times10^9\ \mathrm{N/m^2}}{1.025\times10^3\ \mathrm{kg/m^3}}} = \boxed{2.1\times10^3\ \mathrm{m}}$$

44. (*a*) Both waves travel the same distance, so $\Delta x = v_1 t_1 = v_2 t_2$. We let the smaller speed be v_1, and the larger speed be v_2. The slower wave will take longer to arrive, and so t_1 is more than t_2.

$$t_1 = t_2 + 2.0\ \mathrm{min} = t_2 + 120\ \mathrm{s} \;\rightarrow\; v_1(t_2 + 120\ \mathrm{s}) = v_2 t_2 \;\rightarrow$$

$$t_2 = \frac{v_1}{v_2 - v_1}(120\ \mathrm{s}) = \frac{5.5\ \mathrm{km/s}}{8.5\ \mathrm{km/s} - 5.5\ \mathrm{km/s}}(120\ \mathrm{s}) = 220\ \mathrm{s}$$

$$\Delta x = v_2 t_2 = (8.5\ \mathrm{km/s})(220\ \mathrm{s}) = \boxed{1.9\times10^3\ \mathrm{km}}$$

(*b*) This is not enough information to determine the epicenter. All that is known is the distance of the epicenter from the seismic station. The direction is not known, so the epicenter lies on a circle of radius $1.9\times10^3\ \mathrm{km}$ from the seismic station. Readings from at least two other seismic stations are needed to determine the epicenter's position.

45. We assume that the earthquake wave is moving the ground vertically, since it is a transverse wave. An object sitting on the ground will then be moving with SHM, due to the two forces on it – the normal force upwards from the ground and the weight downwards due to gravity. If the object loses contact with the ground, then the normal force will be zero, and the only force on the object will be its weight. If the only force is the weight, then the object will have an acceleration of *g* downwards. Thus the limiting condition for beginning to lose contact with the ground is when the maximum acceleration caused by the wave is greater than *g*. Any larger downward acceleration and the ground would "fall" quicker than the object. The maximum acceleration is related to the amplitude and the frequency as follows.

$$a_{\max} = \omega^2 A > g \;\rightarrow\; A > \frac{g}{\omega^2} = \frac{g}{4\pi^2 f^2} = \frac{9.8\ \mathrm{m/s^2}}{4\pi^2(0.50\ \mathrm{Hz})^2} = \boxed{0.99\ \mathrm{m}}$$

46. (*a*) Assume that the earthquake waves spread out spherically from the source. Under those conditions, Eq. (11-16b) applies, stating that intensity is inversely proportional to the square of the distance from the source of the wave.

$$I_{20\ \mathrm{km}}/I_{10\ \mathrm{km}} = (10\ \mathrm{km})^2/(20\ \mathrm{km})^2 = \boxed{0.25}$$

(*b*) The intensity is proportional to the square of the amplitude, and so the amplitude is inversely proportional to the distance from the source of the wave.

$$A_{20\ \mathrm{km}}/A_{10\ \mathrm{km}} = 10\ \mathrm{km}/20\ \mathrm{km} = \boxed{0.50}$$

47. (*a*) Assuming spherically symmetric waves, the intensity will be inversely proportional to the square of the distance from the source. Thus Ir^2 will be constant.

$$I_{\text{near}} r_{\text{near}}^2 = I_{\text{far}} r_{\text{far}}^2 \quad \rightarrow$$

$$I_{\text{near}} = I_{\text{far}} \frac{r_{\text{far}}^2}{r_{\text{near}}^2} = \left(2.0\times10^6 \text{ W/m}^2\right)\frac{(48 \text{ km})^2}{(1 \text{ km})^2} = 4.608\times10^9 \text{ W/m}^2 \approx \boxed{4.6\times10^9 \text{ W/m}^2}$$

(*b*) The power passing through an area is the intensity times the area.

$$P = IA = \left(4.608\times10^9 \text{ W/m}^2\right)\left(5.0 \text{ m}^2\right) = \boxed{2.3\times10^{10} \text{ W}}$$

48. From Equation (11-18), if the speed, medium density, and frequency of the two waves are the same, then the intensity is proportional to the square of the amplitude.

$$I_2/I_1 = E_2/E_1 = A_2^2/A_1^2 = 2 \quad \rightarrow \quad A_2/A_1 = \sqrt{2} = \boxed{1.41}$$

The more energetic wave has the larger amplitude.

49. From Equation (11-18), if the speed, medium density, and frequency of the two waves are the same, then the intensity is proportional to the square of the amplitude.

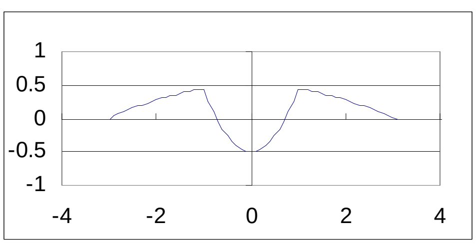
$$I_2/I_1 = P_2/P_1 = A_2^2/A_1^2 = 3 \quad \rightarrow \quad A_2/A_1 = \sqrt{3} = \boxed{1.73}$$

The more energetic wave has the larger amplitude.

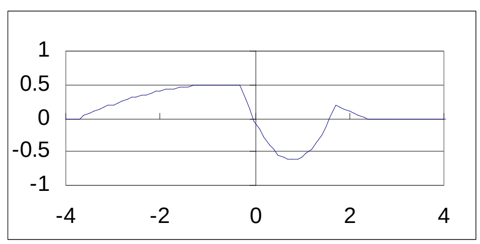
50. The bug moves in SHM as the wave passes. The maximum KE of a particle in SHM is the total energy, which is given by $E_{\text{total}} = \frac{1}{2}kA^2$. Compare the two KE maxima.

$$\frac{KE_2}{KE_1} = \frac{\frac{1}{2}kA_2^2}{\frac{1}{2}kA_1^2} = \left(\frac{A_2}{A_1}\right)^2 = \left(\frac{2.25 \text{ cm}}{3.0 \text{ cm}}\right)^2 = \boxed{0.56}$$

51. (*a*)



(*b*)



(*c*) The energy is all kinetic energy at the moment when the string has no displacement. There is no elastic potential energy at that moment. Each piece of the string has speed but no displacement.

52. The frequencies of the harmonics of a string that is fixed at both ends are given by $f_n = nf_1$, and so the first four harmonics are $\boxed{f_1 = 440 \text{ Hz}, f_2 = 880 \text{ Hz}, f_3 = 1320 \text{ Hz}, f_4 = 1760 \text{ Hz}}$.

53. The fundamental frequency of the full string is given by $f_{\text{unfingered}} = \dfrac{v}{2L} = 294 \text{ Hz}$. If the length is reduced to 2/3 of its current value, and the velocity of waves on the string is not changed, then the new frequency will be

$$f_{\text{fingered}} = \frac{v}{2\left(\frac{2}{3}L\right)} = \frac{3}{2}\frac{v}{2L} = \left(\frac{3}{2}\right) f_{\text{unfingered}} = \left(\frac{3}{2}\right) 294\text{ Hz} = \boxed{441\text{ Hz}}$$

54. Four loops is the standing wave pattern for the 4th harmonic, with a frequency given by $f_4 = 4f_1 = 280\text{ Hz}$. Thus $\boxed{f_1 = 70\text{ Hz}, f_2 = 140\text{ Hz}, f_3 = 210\text{ Hz and } f_5 = 350\text{ Hz}}$ are all other resonant frequencies.

55. Adjacent nodes are separated by a half-wavelength, as examination of Figure 11-40 will show.

$$\lambda = \frac{v}{f} \quad \rightarrow \quad \Delta x_{\text{node}} = \tfrac{1}{2}\lambda = \frac{v}{2f} = \frac{92\text{ m/s}}{2(475\text{ Hz})} = \boxed{9.7\times10^{-2}\text{ m}}$$

56. Since $f_n = nf_1$, two successive overtones differ by the fundamental frequency, as shown below.

$$\Delta f = f_{n+1} - f_n = (n+1)f_1 - nf_1 = f_1 = 350\text{ Hz} - 280\text{ Hz} = \boxed{70\text{ Hz}}$$

57. The speed of waves on the string is given by equation (11-13), $v = \sqrt{\frac{F_T}{m/L}}$. The resonant frequencies of a string with both ends fixed are given by equation (11-19b), $f_n = \frac{nv}{2L_{\text{vib}}}$, where L_{vib} is the length of the portion that is actually vibrating. Combining these relationships allows the frequencies to be calculated.

$$f_n = \frac{n}{2L_{\text{vib}}}\sqrt{\frac{F_T}{m/L}} \qquad f_1 = \frac{1}{2(0.62\text{ m})}\sqrt{\frac{520\text{ N}}{(3.6\times10^{-3}\text{ kg})/(0.90\text{ m})}} = 290.77\text{ Hz}$$

$$f_2 = 2f_1 = 581.54\text{ Hz} \qquad f_3 = 3f_1 = 872.31\text{ Hz}$$

So the three frequencies are $\boxed{290\text{ Hz}, 580\text{ Hz}, 870\text{ Hz}}$, to 2 significant figures.

58. From Equation (11-19b), $f_n = \frac{nv}{2L}$, we see that the frequency is proportional to the wave speed on the stretched string. From equation (11-13), $v = \sqrt{\frac{F_T}{m/L}}$, we see that the wave speed is proportional to the square root of the tension. Thus the frequency is proportional to the square root of the tension.

$$\sqrt{\frac{F_{T2}}{F_{T1}}} = \frac{f_2}{f_1} \quad \rightarrow \quad F_{T2} = \left(\frac{f_2}{f_1}\right)^2 F_{T1} = \left(\frac{200\text{ Hz}}{205\text{ Hz}}\right)^2 F_{T1} = 0.952 F_{T1}$$

Thus the tension should be $\boxed{\text{decreased by 4.8\%}}$.

59. The string must vibrate in a standing wave pattern to have a certain number of loops. The frequency of the standing waves will all be 60 Hz, the same as the vibrator. That frequency is also expressed by Equation (11-19b), $f_n = \frac{nv}{2L}$. The speed of waves on the string is given by Equation (11-13),

$v=\sqrt{\frac{F_T}{m/L}}$. The tension in the string will be the same as the weight of the masses hung from the end of the string, $F_T = mg$. Combining these relationships gives an expression for the masses hung from the end of the string.

(*a*) $f_n = \frac{nv}{2L} = \frac{n}{2L}\sqrt{\frac{F_T}{m/L}} = \frac{n}{2L}\sqrt{\frac{mg}{(m/L)}} \rightarrow m = \frac{4L^2 f_n^2 (m/L)}{n^2 g}$

$$m_1 = \frac{4(1.50\text{ m})^2 (60\text{ Hz})^2 (3.9\times10^{-4}\text{ kg/m})}{1^2 (9.80\text{ m/s}^2)} = 1.289\text{ kg} \approx \boxed{1.3\text{ kg}}$$

(*b*) $m_2 = \frac{m_1}{2^2} = \frac{1.289\text{ kg}}{4} = \boxed{0.32\text{ kg}}$

(*c*) $m_5 = \frac{m_1}{5^2} = \frac{1.289\text{ kg}}{25} = \boxed{5.2\times10^{-2}\text{ kg}}$

60. The tension in the string is the weight of the hanging mass, $F_T = mg$. The speed of waves on the string can be found by $v=\sqrt{\frac{F_T}{m/L}}=\sqrt{\frac{mg}{(m/L)}}$, and the frequency is given as $f = 60\text{ Hz}$. The wavelength of waves created on the string will thus be given by

$$\lambda = \frac{v}{f} = \frac{1}{f}\sqrt{\frac{mg}{(m/L)}} = \frac{1}{60\text{ Hz}}\sqrt{\frac{(0.080\text{ kg})(9.80\text{ m/s}^2)}{(3.9\times10^{-4}\text{ kg/m})}} = 0.7473\text{ m}.$$

The length of the string must be an integer multiple of half of the wavelength for there to be nodes at both ends and thus form a standing wave. Thus $L = \lambda/2, \lambda, 3\lambda/2, \cdots$, and so on. This gives $L = 0.37\text{ m}, 0.75\text{ m}, 1.12\text{ m}, 1.49\text{ m}$ as the possible lengths, and so there are $\boxed{4}$ standing wave patterns that may be achieved.

61. From the description of the water's behavior, there is an anti-node at each end of the tub, and a node in the middle. Thus one wavelength is twice the tube length.

$$v = \lambda f = (2L_{tub}) f = 2(0.65\text{ m})(0.85\text{ Hz}) = \boxed{1.1\text{ m/s}}$$

62. The speed in the second medium can be found from the law of refraction, Equation (11-20).

$$\frac{\sin\theta_2}{\sin\theta_1} = \frac{v_2}{v_1} \rightarrow v_2 = v_1\frac{\sin\theta_2}{\sin\theta_1} = (8.0\text{ km/s})\left(\frac{\sin 35^\circ}{\sin 47^\circ}\right) = \boxed{6.3\text{ km/s}}$$

63. The angle of refraction can be found from the law of refraction, Equation (11-20).

$$\frac{\sin\theta_2}{\sin\theta_1} = \frac{v_2}{v_1} \rightarrow \sin\theta_2 = \sin\theta_1\frac{v_2}{v_1} = \sin 34^\circ\frac{2.1\text{ m/s}}{2.8\text{ m/s}} = 0.419 \rightarrow \theta_2 = \sin^{-1} 0.419 = \boxed{25^\circ}$$

64. The angle of refraction can be found from the law of refraction, Equation (11-20). The relative velocities can be found from the relationship given in the problem.

$$\frac{\sin\theta_2}{\sin\theta_1}=\frac{v_2}{v_1}=\frac{331+0.60T_2}{331+0.60T_1} \rightarrow \sin\theta_2=\sin 25^\circ\frac{331+0.60(-10)}{331+0.60(10)}=\sin 25^\circ\frac{325}{337}=0.4076$$

$$\theta_2=\sin^{-1}0.4076=\boxed{24^\circ}$$

65. The angle of refraction can be found from the law of refraction, Equation (11-20). The relative velocities can be found from Equation (11-14a).

$$\frac{\sin\theta_2}{\sin\theta_1}=\frac{v_2}{v_1}=\frac{\sqrt{E/\rho_2}}{\sqrt{E/\rho_1}}=\sqrt{\frac{\rho_1}{\rho_2}}=\sqrt{\frac{SG_1\rho_{\text{water}}}{SG_2\rho_{\text{water}}}}=\sqrt{\frac{SG_1}{SG_2}}$$

$$\sin\theta_2=\sin\theta_1\sqrt{\frac{SG_1}{SG_2}}=\sin 38^\circ\sqrt{\frac{3.6}{2.8}}=0.70 \rightarrow \theta_2=\sin^{-1}0.70=\boxed{44^\circ}$$

66. The error of 2° is allowed due to diffraction of the waves. If the waves are incident at the "edge" of the dish, they can still diffract into the dish if the relationship $\theta\approx\lambda/L$ is satisfied.

$$\theta\approx\frac{\lambda}{L} \rightarrow \lambda=L\theta=(0.5\text{ m})\left(2^\circ\times\frac{\pi\text{ rad}}{180^\circ}\right)=1.745\times10^{-2}\text{ m}\approx\boxed{2\times10^{-2}\text{ m}}$$

If the wavelength is longer than that, there will not be much diffraction, but "shadowing" instead.

67. The unusual decrease of water corresponds to a trough in Figure 11-24. The crest or peak of the wave is then one-half wavelength distant. The peak is 125 km away, traveling at 750 km/hr.

$$\Delta x=vt \rightarrow t=\frac{\Delta x}{v}=\frac{125\text{ km}}{750\text{ km/hr}}\left(\frac{60\text{ min}}{1\text{ hr}}\right)=\boxed{10\text{ min}}$$

68. Apply the conservation of mechanical energy to the car, calling condition # 1 to be before the collision and condition # 2 to be after the collision. Assume that all of the kinetic energy of the car is converted to potential energy stored in the bumper. We know that $x_1=0$ and $v_2=0$.

$$E_1=E_2 \rightarrow \tfrac{1}{2}mv_1^2+\tfrac{1}{2}kx_1^2=\tfrac{1}{2}mv_2^2+\tfrac{1}{2}kx_2^2 \rightarrow \tfrac{1}{2}mv_1^2=\tfrac{1}{2}kx_2^2 \rightarrow$$

$$x_2=\sqrt{\frac{m}{k}}v_1=\sqrt{\frac{1500\text{ kg}}{550\times10^3\text{ N/m}}}(2.2\text{ m/s})=\boxed{0.11\text{ m}}$$

69. Consider the conservation of energy for the person. Call the unstretched position of the fire net the zero location for both elastic potential energy and gravitational potential energy. The amount of stretch of the fire net is given by *x*, measured positively in the downward direction. The vertical displacement for gravitational potential energy is given by the variable *y*, measured positively for the upward direction. Calculate the spring constant by conserving energy between the window height and the lowest location of the person. The person has no kinetic energy at either location.

$$E_{\text{top}}=E_{\text{bottom}} \rightarrow mgy_{\text{top}}=mgy_{\text{bottom}}+\tfrac{1}{2}kx_{\text{bottom}}^2$$

$$k=2mg\frac{(y_{top}-y_{\text{bottom}})}{x_{\text{bottom}}^2}=2(65\text{ kg})(9.8\text{ m/s}^2)\frac{[18\text{ m}-(-1.1\text{ m})]}{(1.1\text{ m})^2}=2.011\times10^4\text{ N/m}$$

(*a*) If the person were to lie on the fire net, they would stretch the net an amount such that the upward force of the net would be equal to their weight.

$$F = kx = mg \quad \rightarrow \quad x = \frac{mg}{k} = \frac{(65\text{ kg})(9.8\text{ m/s}^2)}{2.011\times10^4\text{ N/m}} = \boxed{3.2\times10^{-2}\text{ m}}$$

(*b*) To find the amount of stretch given a starting height of 35 m, again use conservation of energy. Note that $y_{\text{bottom}} = -x$, and there is no kinetic energy at the top or bottom positions.

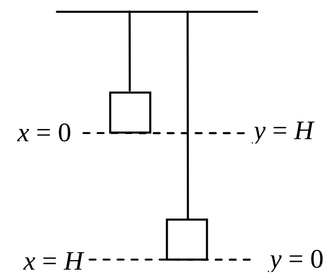
$$E_{\text{top}} = E_{\text{bottom}} \quad \rightarrow \quad mgy_{\text{top}} = mgy_{\text{bottom}} + \tfrac{1}{2}kx^2 \quad \rightarrow \quad x^2 - 2\frac{mg}{k}x - 2\frac{mg}{k}y_{top} = 0$$

$$x^2 - 2\frac{(65\text{ kg})(9.8\text{ m/s}^2)}{2.011\times10^4\text{ N/m}}x - 2\frac{(65\text{ kg})(9.8\text{ m/s}^2)}{2.011\times10^4\text{ N/m}}(35\text{ m}) = 0 \quad \rightarrow$$

$$x^2 - 0.06335x - 2.2173 = 0 \quad \rightarrow \quad x = 1.5211\text{ m},\ -1.458\text{ m}$$

This is a quadratic equation. The solution is the positive root, since the net must be below the unstretched position. The result is $\boxed{1.5\text{ m}}$.

70. Consider energy conservation for the mass over the range of motion from "letting go" (the highest point) to the lowest point. The mass falls the same distance that the spring is stretched, and has no KE at either endpoint. Call the lowest point the zero of gravitational potential energy. The variable "*x*" represents the amount that the spring is stretched from the equilibrium position.



$$E_{\text{top}} = E_{\text{bottom}}$$

$$\tfrac{1}{2}mv_{\text{top}}^2 + mgy_{\text{top}} + \tfrac{1}{2}kx_{\text{top}}^2 = \tfrac{1}{2}mv_{\text{bottom}}^2 + mgy_{\text{bottom}} + \tfrac{1}{2}kx_{\text{bottom}}^2$$

$$0 + mgH + 0 = 0 + 0 + \tfrac{1}{2}kH^2 \quad \rightarrow \quad \frac{k}{m} = \frac{2g}{H} = \omega^2 \quad \rightarrow \quad \omega = \sqrt{\frac{2g}{H}}$$

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{2g}{H}} = \frac{1}{2\pi}\sqrt{\frac{2(9.8\text{ m/s}^2)}{0.33\text{ m}}} = \boxed{1.2\text{ Hz}}$$

71. (*a*) From conservation of energy, the initial kinetic energy of the car will all be changed into elastic potential energy by compressing the spring.

$$E_1 = E_2 \quad \rightarrow \quad \tfrac{1}{2}mv_1^2 + \tfrac{1}{2}kx_1^2 = \tfrac{1}{2}mv_2^2 + \tfrac{1}{2}kx_2^2 \quad \rightarrow \quad \tfrac{1}{2}mv_1^2 = \tfrac{1}{2}kx_2^2 \quad \rightarrow$$

$$k = m\frac{v_1^2}{x_2^2} = (950\text{ kg})\frac{(22\text{ m/s})^2}{(5.0\text{ m})^2} = 1.8392\times10^4\text{ N/m} \approx \boxed{1.8\times10^4\text{ N/m}}$$

(*b*) The car will be in contact with the spring for half a period, as it moves from the equilibrium location to maximum displacement and back to equilibrium.

$$\tfrac{1}{2}T = \tfrac{1}{2}2\pi\sqrt{\frac{m}{k}} = \pi\sqrt{\frac{(950\text{ kg})}{1.8392\times10^4\text{ N/m}}} = \boxed{0.71\text{ s}}$$

72. The frequency at which the water is being shaken is about 1 Hz. The sloshing coffee is in a standing wave mode, with anti-nodes at each edge of the cup. The cup diameter is thus a half-wavelength, or $\lambda = 16\text{ cm}$. The wave speed can be calculated from the frequency and the wavelength.

$$v = \lambda f = (16\text{ cm})(1\text{ Hz}) = \boxed{16\text{ cm/s}}$$

73. Relative to the fixed needle position, the ripples are moving with a linear velocity given by

$$v = \left(33\frac{\text{rev}}{\text{min}}\right)\left(\frac{1\,\text{min}}{60\ \text{s}}\right)\left(\frac{2\pi\left(0.108\ \text{m}\right)}{1\ \text{rev}}\right) = 0.373\,\text{m/s}$$

This speed is the speed of the ripple waves moving past the needle. The frequency of the waves is

$$f = \frac{v}{\lambda} = \frac{0.373\,\text{m/s}}{1.70\times10^{-3}\,\text{m}} = \boxed{220\ \text{Hz}}$$

74. The equation of motion is $x = 0.650\cos 7.40\,t = A\cos\omega t$.

(*a*) The amplitude is $A = \boxed{0.650\ \text{m}}$

(*b*) The frequency is given by $\omega = 2\pi f = 7.40\,\text{rad/s} \;\rightarrow\; f = \frac{7.40\,\text{rad/s}}{2\pi\ \text{rad}} = 1.177\,\text{Hz} \approx \boxed{1.18\ \text{Hz}}$

(*c*) The total energy is given by

$$E_{\text{total}} = \tfrac{1}{2}kA^2 = \tfrac{1}{2}m\omega^2A^2 = \tfrac{1}{2}\left(2.00\,\text{kg}\right)\left(7.40\,\text{rad/s}\right)^2\left(0.650\ \text{m}\right)^2 = 23.136\,\text{J} \approx \boxed{23.1\ \text{J}}.$$

(*d*) The potential energy is found by

$$PE = \tfrac{1}{2}kx^2 = \tfrac{1}{2}m\omega^2x^2 = \tfrac{1}{2}\left(2.00\,\text{kg}\right)\left(7.40\,\text{rad/s}\right)^2\left(0.260\ \text{m}\right)^2 = 3.702\,\text{J} \approx \boxed{3.70\ \text{J}}.$$

The kinetic energy is found by

$$KE = E_{\text{total}} - PE = 23.136\ \text{J} - 3.702\ \text{J} = \boxed{19.4\ \text{J}}.$$

75. The frequency of a simple pendulum is given by $f = \frac{1}{2\pi}\sqrt{\frac{g}{L}}$. The pendulum is accelerating vertically which is equivalent to increasing (or decreasing) the acceleration due to gravity by the acceleration of the pendulum.

(*a*) $$f_{\text{new}} = \frac{1}{2\pi}\sqrt{\frac{g+a}{L}} = \frac{1}{2\pi}\sqrt{\frac{1.50g}{L}} = \sqrt{1.50}\,\frac{1}{2\pi}\sqrt{\frac{g}{L}} = \sqrt{1.50}\,f = \boxed{1.22\,f}$$

(*b*) $$f_{\text{new}} = \frac{1}{2\pi}\sqrt{\frac{g+a}{L}} = \frac{1}{2\pi}\sqrt{\frac{0.5g}{L}} = \sqrt{0.5}\,\frac{1}{2\pi}\sqrt{\frac{g}{L}} = \sqrt{0.5}\,f = \boxed{0.71f}$$

76. The force of the man's weight causes the raft to sink, and that causes the water to put a larger upward force on the raft. This extra buoyant force is a restoring force, because it is in the opposite direction of the force put on the raft by the man. This is analogous to pulling down on a mass-spring system that is in equilibrium, by applying an extra force. Then when the man steps off, the restoring force pushes upward on the raft, and thus the raft – water system acts like a spring, with a spring constant found as follows.

$$k = \frac{F}{x} = \frac{\left(75\ \text{kg}\right)\left(9.8\,\text{m/s}^2\right)}{4.0\times10^{-2}\,\text{m}} = 1.8375\times10^4\ \text{N/m}$$

(*a*) The frequency of vibration is determined by the spring constant and the mass of the raft.

$$f_n = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{1.8375\times10^4\ \text{N/m}}{220\ \text{kg}}} = 1.455\,\text{Hz} \approx \boxed{1.5\ \text{Hz}}$$

(*b*) As explained in the text, for a vertical spring the gravitational potential energy can be ignored if the displacement is measured from the oscillator's equilibrium position. The total energy is thus

$$E_{\text{total}} = \tfrac{1}{2}kA^2 = \tfrac{1}{2}\left(1.8375\times10^4\ \text{N/m}\right)\left(4.0\times10^{-2}\,\text{m}\right)^2 = 14.7\,\text{J} \approx \boxed{15\ \text{J}}.$$

77. (*a*) The overtones are given by $f_n = nf_1, n = 2,3,4\ldots$

$$\text{G}: \quad f_2 = 2(392\text{ Hz}) = \boxed{784\text{ Hz}} \quad f_3 = 3(392\text{ Hz}) = \boxed{1180\text{ Hz}}$$

$$\text{A}: \quad f_2 = 2(440\text{ Hz}) = \boxed{880\text{ Hz}} \quad f_3 = 3(440\text{ Hz}) = \boxed{1320\text{ Hz}}$$

(*b*) If the two strings have the same length, they have the same wavelength. The frequency difference is then due to a difference in wave speed caused by different masses for the strings.

$$\frac{f_G}{f_\text{A}} = \frac{v_G/\lambda}{v_A/\lambda} = \frac{v_G}{v_A} = \frac{\sqrt{\dfrac{F_\text{T}}{m_G/L}}}{\sqrt{\dfrac{F_\text{T}}{m_A/L}}} = \sqrt{\frac{m_A}{m_G}} \rightarrow \frac{m_G}{m_A} = \left(\frac{f_A}{f_G}\right)^2 = \left(\frac{440}{392}\right)^2 = \boxed{1.26}$$

(*c*) If the two strings have the same mass per unit length and the same tension, then the wave speed on both strings is the same. The frequency difference is then due to a difference in wavelength. For the fundamental, the wavelength is twice the length of the string.

$$\frac{f_G}{f_\text{A}} = \frac{v/\lambda_G}{v/\lambda_A} = \frac{\lambda_A}{\lambda_G} = \frac{2L_A}{2L_G} \rightarrow \frac{L_G}{L_A} = \frac{f_A}{f_G} = \frac{440}{392} = \boxed{1.12}$$

(*d*) If the two strings have the same length, they have the same wavelength. The frequency difference is then due to a difference in wave speed caused by different tensions for the strings.

$$\frac{f_G}{f_\text{A}} = \frac{v_G/\lambda}{v_A/\lambda} = \frac{v_G}{v_A} = \frac{\sqrt{\dfrac{F_{\text{T}G}}{m/L}}}{\sqrt{\dfrac{F_{\text{T}A}}{m/L}}} = \sqrt{\frac{F_{\text{T}G}}{F_{\text{T}A}}} \rightarrow \frac{F_{\text{T}G}}{F_{\text{T}AA}} = \left(\frac{f_G}{f_A}\right)^2 = \left(\frac{392}{440}\right)^2 = \boxed{0.794}$$

78. (*a*) Since the cord is not accelerating to the left or right, the tension in the cord must be the same everywhere. Thus the tension is the same in the two parts of the cord. The speed difference will then be due to the different mass densities of the two parts of the cord. Let the symbol μ represent the mass per unit length of each part of the cord.

$$\frac{v_\text{H}}{\text{v}_\text{L}} = \frac{\left(\sqrt{F_\text{T}/\mu}\right)_\text{H}}{\left(\sqrt{F_\text{T}/\mu}\right)_\text{L}} = \boxed{\sqrt{\frac{\mu_L}{\mu_H}}}$$

(*b*) The wavelength ratio is found as follows.

$$\frac{\lambda_\text{H}}{\lambda_\text{L}} = \frac{(v/f)_\text{H}}{(v/f)_\text{L}} = \frac{v_\text{H}}{v_\text{L}} = \boxed{\sqrt{\frac{\mu_L}{\mu_H}}}$$

The two frequencies must be the same for the cord to remain continuous at the boundary. If the two parts of the cord oscillate at different frequencies, the cord cannot stay in one piece, because the two parts would be out of phase with each other at various times.

(*c*) Since $\mu_\text{H} > \mu_\text{L}$, we see that $\lambda_\text{H} < \lambda_\text{L}$, and so the wavelength is greater in the $\boxed{\text{lighter cord}}$.

79. (*a*) The maximum speed is given by

$$v_\text{max} = 2\pi f\, A = 2\pi(264\text{ Hz})(1.8\times10^{-3}\text{m}) = \boxed{3.0\,\text{m/s}}.$$

(*b*) The maximum acceleration is given by

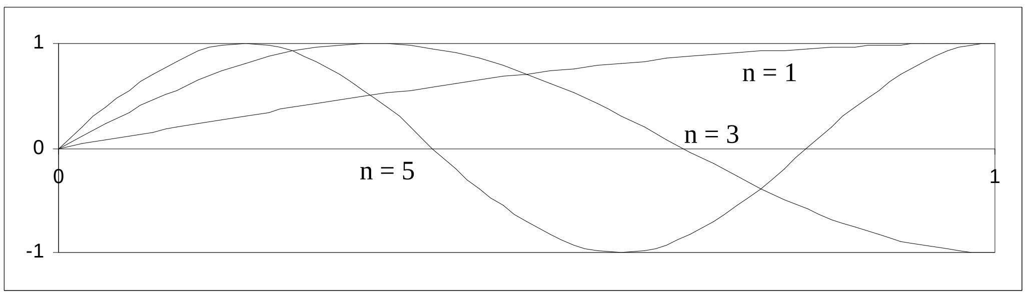
$$a_\text{max} = 4\pi^2 f^2 A = 4\pi^2 (264\text{ Hz})^2 (1.8\times10^{-3}\text{m}) = \boxed{5.0\times10^3\text{ m/s}^2}.$$

80. For the pebble to lose contact with the board means that there is no normal force of the board on the pebble. If there is no normal force on the pebble, then the only force on the pebble is the force of gravity, and the acceleration of the pebble will be *g* downward, the acceleration due to gravity. This is the maximum downward acceleration that the pebble can have. Thus if the board's downward acceleration exceeds *g*, then the pebble will lose contact. The maximum acceleration and the amplitude are related by $a_{max} = 4\pi^2 f^2 A$.

$$a_{max} = 4\pi^2 f^2 A \le g \quad \to \quad A \le \frac{g}{4\pi^2 f^2} \le \frac{9.8\,\text{m/s}^2}{4\pi^2 (1.5\text{ Hz})^2} \le \boxed{1.1\times10^{-1}\text{m}}$$

81. For a resonant condition, the free end of the string will be an antinode, and the fixed end of the string will be a node. The minimum distance from a node to an antinode is $\lambda/4$. Other wave patterns that fit the boundary conditions of a node at one end and an antinode at the other end include $3\lambda/4$, $5\lambda/4$, … . See the diagrams. The general relationship is $L = (2n-1)\lambda/4$, $n = 1, 2, 3, \cdots$.

Solving for the wavelength gives $\boxed{\lambda = \dfrac{4L}{2n-1}\ ,\ n = 1, 2, 3, \cdots}$



82. The period of a pendulum is given by $T = 2\pi\sqrt{L/g}$, and so the length is $L = \dfrac{T^2 g}{4\pi^2}$.

(*a*) $$L_{\text{Austin}} = \frac{T^2 g_{\text{Austin}}}{4\pi^2} = \frac{(2.000\text{ s})^2 (9.793\,\text{m/s}^2)}{4\pi^2} = \boxed{0.9922\text{ m}}$$

(*b*) $$L_{\text{Paris}} = \frac{T^2 g_{\text{Paris}}}{4\pi^2} = \frac{(2.000\text{ s})^2 (9.809\,\text{m/s}^2)}{4\pi^2} = 0.9939\text{ m}$$

$$L_{\text{Paris}} - L_{\text{Austin}} = 0.9939\text{ m} - 0.9922\text{ m} = 0.0016\text{ m} = \boxed{1.6\text{ mm}}$$

(*c*) $$L_{\text{Moon}} = \frac{T^2 g_{\text{Moon}}}{4\pi^2} = \frac{(2.00\text{ s})^2 (1.62\,\text{m/s}^2)}{4\pi^2} = \boxed{0.164\text{ m}}$$

83. The spring, originally of length l_0, will be stretched downward to a new equilibrium length L when the mass is hung on it. The amount of downward stretch $L - l_0$ is found from setting the spring force upward on the mass equal to the weight of the mass: $k(L - l_0) = mg \quad \to \quad L = l_0 + mg/k$. The length of the pendulum is then $L = l_0 + mg/k$. The period of the vertical oscillations is given by $T_{\text{ver}} = 2\pi\sqrt{m/k}$, while the period of the pendulum oscillations is given by $T_{\text{pen}} = 2\pi\sqrt{L/g}$. Now compare the periods of the two motions.

$$\frac{T_{\text{pen}}}{T_{\text{ver}}}=\frac{2\pi\sqrt{\left(l_0+mg/k\right)/g}}{2\pi\sqrt{m/k}}=\sqrt{\frac{l_0+mg/k}{mg/k}}=\sqrt{1+\frac{l_0k}{mg}}>1\ \rightarrow$$

$$\boxed{T_{\text{pen}}>T_{\text{ver}}\text{, by a factor of }\sqrt{1+\frac{l_0k}{mg}}}$$

84. Block *m* stays on top of block *M* (executing SHM relative to the ground) without slipping due to static friction. The maximum static frictional force on *m* is $F_{\text{fr}_{\max}}=\mu_s mg$. This frictional force causes block *m* to accelerate, so $ma_{\max}=\mu_s mg\ \rightarrow\ a_{\max}=\mu_s g$. Thus for the blocks to stay in contact without slipping, the maximum acceleration of block *M* is also $a_{\max}=\mu_s g$. But an object in SHM has a maximum acceleration given by $a_{\max}=\omega^2 A=\frac{k}{M_{total}}A$. Equate these two expressions for the maximum acceleration.

$$a_{\max}=\frac{k}{M_{total}}A=\mu_s g\ \rightarrow\ A=\frac{\mu_s g}{k}\left(M+m\right)=\frac{(0.30)\left(9.8\,\text{m/s}^2\right)}{130\,\text{N/m}}(6.25\text{ kg})=\boxed{0.14\text{ m}}$$

85. The speed of the pulses is found from the tension and mass per unit length of the wire.

$$v=\sqrt{\frac{F_{\text{T}}}{m/L}}=\sqrt{\frac{255\text{ N}}{(0.123\text{ kg})/(10.0\text{ m})}}=143.985\,\text{m/s}$$

The total distance traveled by the two pulses will be the length of the wire. The second pulse has a shorter time of travel than the first pulse, by 20.0 ms.

$$L=d_1+d_2=vt_1+vt_2=vt_1+v\left(t_1-2.00\times10^{-2}\right)$$

$$t_1=\frac{L+2.00\times10^{-2}v}{2v}=\frac{(10.0\text{ m})+2.00\times10^{-2}(143.985\,\text{m/s})}{2(143.985\,\text{m/s})}=4.4726\times10^{-2}\text{s}$$

$$d_1=vt_1=(143.985\,\text{m/s})\left(4.4726\times10^{-2}\text{s}\right)=6.44\text{ m}$$

The two pulses meet $\boxed{6.44\text{ m}}$ from the end where the first pulse originated.

86. For the penny to stay on the block at all times means that there will be a normal force on the penny from the block, exerted upward. If down is taken to be the positive direction, then the net force on the penny is $F_{\text{net}}=mg-F_N=ma$. Solving for the magnitude of the normal force gives $F_N=mg-ma$. This expression is always positive if the acceleration is upwards ($a<0$), and so there is no possibility of the penny losing contact while accelerating upwards. But if a downward acceleration were to be larger than *g*, then the normal force would go to zero, since the normal force cannot switch directions $\left(F_N>0\right)$. Thus the limiting condition is $a_{\text{down}}=g$. This is the maximum value for the acceleration. For SHM, we also know that $a_{\max}=\omega^2 A=\frac{k}{M+m}A\approx\frac{k}{M}A$. Equate these two values for the acceleration.

$$a_{\max}=\frac{k}{M}A=g\ \rightarrow\ \boxed{A=\frac{Mg}{k}}$$

87. The car on the end of the cable produces tension in the cable, and stretches the cable according to Equation (9-4), $\Delta L = \frac{1}{E}\frac{F}{A}L_o$, where E is Young's modulus. Rearrange this equation to see that the tension force is proportional to the amount of stretch, $F = \frac{EA}{L_o}\Delta L$, and so the effective spring constant is $k = \frac{EA}{L_o}$. The period of the bouncing can be found from the spring constant and the mass on the end of the cable.

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{mL_o}{EA}} = 2\pi\sqrt{\frac{(1200\text{ kg})(22\text{ m})}{(200\times10^{9}\text{ N/m}^2)\,\pi\,(3.2\times10^{-3}\text{m})^2}} = \boxed{0.40\text{ s}}$$

88. From Equation (9-6) and Figure (9-22c), the restoring force on the top of the Jell-O is $F = \frac{GA}{L_o}\Delta L$, and is in the opposite direction to the displacement of the top from the equilibrium condition. Thus the "spring constant" for the restoring force is $k = \frac{GA}{L_o}$. If you were to look at a layer of Jell-O closer to the base, the displacement would be less, but so would the restoring force in proportion, and so we estimate all of the Jell-O as having the same spring constant. The frequency of vibration can be determined from the spring constant and the mass of the Jell-O.

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{GA/L_o}{\rho V}} = \frac{1}{2\pi}\sqrt{\frac{GA/L_o}{\rho A L_o}} = \frac{1}{2\pi}\sqrt{\frac{G}{\rho L_o^2}}$$

$$= \frac{1}{2\pi}\sqrt{\frac{520\,\text{N/m}^2}{(1300\,\text{kg/m}^3)(4.0\times10^{-2}\,\text{m})^2}} = \boxed{2.5\text{ Hz}}$$